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LONGITUDINAL RADIATION TRANSFER IN MOVEMENT OF A DUST CONTAINING MEDIUM IN A PLANE CHANNEL

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The article presents the solution of the problem of radiative and convective heat exchange in stabilized flow of a scattering medium in a plane channel, with the longitudinal radiative fluxes taken into account.

In radiative and convective heat exchange in short channels [1], and also in channels with abrupt changes of the wall temperature [2, 3], the transfer of radiation along the flow of the medium is considerable. For instance, in practice, the problem of shielding the radiation of the hot inlet end face of a channel from its cooling walls is encountered. To determine the effectiveness of shielding, experiments are needed, but also mathematical modeling of the radiative and convective heat exchange.

Radiative and convective heat exchange in a circular cylindrical channel with a view to the longitudinal radiative fluxes was dealt with in [1-6]. The authors assumed that the flux was hydrodynamically stabilized, and that the flow was both laminar [1-5] and turbulent [1-6]. The authors of [4, 6] took the selectivity of the optical properties of the medium into account, the authors of [5] took scattering of the radiation into account. For a plane channel the problem was solved in [1], and it was assumed there that the medium is nonscattering and the flow laminar.

Let us examine heat exchange in stabilized flow of a radiating, absorbing, anisotropically scattering, heat conducting medium in a plane channel with black walls (Fig. 1). We regard the medium as nonselective. We neglect heat conduction along the flow. Mathematically the problem is formulated in the form of an equation of energy

$$\varphi \frac{\partial \theta}{\partial \bar{x}} = \frac{2}{\operatorname{Pe}} \frac{\partial}{\partial \bar{y}} A \frac{\partial \theta}{\partial \bar{y}} - \frac{H}{\rho c_p \langle v \rangle T_0} \operatorname{div} q_R \tag{1}$$

with the boundary conditions

$$\theta(\bar{y}) = 1, \quad \bar{x} = 0, \tag{2}$$

$$\theta(\bar{y}=0) = \theta(\bar{y}=1) = \theta_1, \ 0 < \bar{x} \leq \bar{a},$$
(3)

$$\theta(\bar{y}=0) = \theta(\bar{y}=1) = \theta_2, \ \bar{a} < \bar{x} < \bar{L},$$
(4)

$$\theta(\overline{y}) = \theta_2, \ \overline{x} = \overline{L}. \tag{5}$$

The following notation was used: $\theta = T/T_0$, $\theta_1 = T_1/T_0$, $\theta_2 = T_2/T_0$, $\bar{x} = x/H$, $\bar{y} = y/H$, $\bar{a} = a/H$, $\bar{L} = L/H$, $\phi = v/\langle v \rangle$, $Pe = 2H\rho c_D \langle v \rangle /\lambda$, $A = 1 + \lambda_S/\lambda$.

The expression for the divergence of the flow density vector of the radiation has the form [7]

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Fig. 1. Diagram to the statement of the problem.

Fig. 2. Dependence of $Q = \langle q \rangle / \rho c_p \langle v \rangle T_0$ on \bar{a} (1) and on τ_0 (2) for $\bar{x} = 0$, $\bar{L} = 4$, $\theta_1 = 0.933$; $\theta_2 = 0.333$; $\phi = 1$, Pe = 4.25 $\cdot 10^4$; A = 30; $\gamma = 0$; $1 - \tau_0 = 0.3$; $2 - \bar{a} = 1.0$.

div
$$\mathbf{q}_{\mathrm{R}} = k (1 - \gamma) (4\sigma T^4 - \int_{4\pi} Id \Omega).$$
 (6)

The radiation intensity is determined from the transfer equation [7]. In the case of a scattering medium it is an integrodifferential equation, and its solution entails considerable mathematical difficulties. However, when a quasiunidimensional approximation [5] is used, the solutions for scattering and nonscattering media are analogous:

$$I(S, \overline{\omega}) = I(0, \overline{\omega}) D(S, S_0) + \frac{k(1-\gamma)}{\pi} \int_0^S \sigma T^4 D(S, S_0) dS$$
(7)

and they differ only by the form of the function of radiation transmission. In the calculations the form of the dependence for $D(S, S_0)$ was taken from [5]. For a nonscattering medium the function of transmission does not depend on $S_0:D(S) = \exp(-kS)$.

When we substitute (6), (7) into (1) and solve the equation of energy numerically, we obtain the temperature distribution in the medium. We used the method of [8] in combination with the iteration method. In the initial iteration the calculation was carried out according to the temperature field found from the solution of the stated problem without taking the longitudinal radiative flows into account [8]. In each subsequent iteration the divergence of the radiative flow was determined in the form of a linear combination of the divergences obtained in the two preceding iterations. The calculation was discontinued when the difference between the mean mass temperatures in all the examined sections for two successive iterations did not exceed 1%. Then from the known temperature field we calculated the local and integral heat flows. The results of the numerical solution for laminar flow are in satisfactory agreement with [1].

In the actual calculations the values of the parameters were taken for the following case. In the arrangement of a metallurgical furnace and a waste-heat boiler, the boiler sometimes has to be mounted above the furnace. On the one hand this entails undesirable heat losses by radiation from the working space of the furnace onto the cold boiler walls. These losses can be reduced by shielding the longitudinal radiant flows: by using the heat insulation of the walls of the initial section of the channel and by changing the radiative properties of the medium. In the adopted model, the plane channel in which a dust containing gas moves simulates the waste-heat boiler, and the radiation by the permeable hot endface wall of the channel simulates the radiation from the working space of the furnace.

The results of the calculations are presented in Figs. 2-4. It follows from Fig. 2 that the heat insulation of one channel groove alone reduces the longitudinal radiative flux by more than half. With $\tau_0 = \text{const}$ and $\gamma < 0.7$, the longitudinal radiative flow is practically independent of γ , and with $\tau_0(1 - \gamma) = \text{const}$ the longitudinal flow decreases monotonically and considerably with increasing γ (Fig. 4). Thus, shielding of the radiation of the inlet end face of the channel is attained most effectively by the heat insulation of the initial section of the channel and by increasing the scattering coefficient of the medium with constant absorption coefficient.



Fig. 3. Change of Q (1) and $\bar{\theta}$ (2) along the channel for $\bar{L} = 4$, $\bar{a} = 2$, $\tau_0 = 0.3$; $\theta_1 = 0.933$; $\theta_2 = 0.333$; $\phi = 1$, Pe = 4.25 \cdot 10⁴; A = 30, $\gamma = 0$.

Fig. 4. Effect of γ on Q for x = 0, $\tilde{L} = 2$, a = 0, $\theta_2 = 0.2$; $\phi = 1$, Pe = 4.25 $\cdot 10^4$; A = 30; $1 - \tau_0 = 1.0$; $2 - \tau_0(1 - \gamma) = 1.0$.

For engineering evaluations of the resulting longitudinal radiant flows in case of a nonscattering medium, the authors obtained the dependence

$$Q' = 1 - (1 - A_1 e^{-2\tau_1}) (1 - e^{-\tau_1}) - A_1 e^{-\tau_1} \theta_1^4 - (1 - A_1) e^{-\tau_1} \int_{\tau_1}^{\tau_2} \theta_1^4 e^{-\tau} d\tau,$$
(8)

where

$$\begin{split} A_{1} &= (1 + \overline{a} - \sqrt{1 + \overline{a^{2}}}); \ \tau_{1} &= 2\tau_{0} \frac{\overline{a}}{1 + \overline{a}}; \\ \tau_{2} &= 2\tau_{0} \frac{\overline{L}}{1 + \overline{L}}; \quad \theta = \left(1 + \frac{3\varepsilon \left(1 + \overline{a}\right)\tau}{\tau_{0} \text{Bo}}\right)^{-1/3}; \\ \varepsilon &= 1 - 2E_{3} \left(\tau_{0}\right); \\ \text{Bo} &= \frac{\rho c_{p} \left\langle \upsilon \right\rangle}{\sigma T_{0}^{3}}; \ Q' &= \frac{\left\langle q \right\rangle}{\sigma T_{0}^{4}}; \end{split}$$

 $E_3(\tau_0)$ is an integroexponential function [7].

Dependence (8) was obtained on the basis of M. T. Smirnov's unidimensional schema [9] with correction of the coefficients according to the numerical solution. The error of (8) does not exceed 20% in the examined range of values of the parameters.

NOTATION

T, temperature of the medium; T_0 , temperature of the medium at the channel entrance; T_1 , wall temperature between the channel entrance and the section with the coordinate x = a; T_2 , wall temperature between the section with the coordinate x = a and the outlet section of the channel; x, longitudinal coordinate; y, transverse coordinate; H, channel height; L, channel length; v, $\langle v \rangle$, local and mean mass velocity, respectively, of the flow; ρ , c_p , λ , λ_s , density, specific isobaric heat capacity, molecular and turbulent heat conduction of the medium, respectively; q_R , density vector of the radiative flow; k, coefficient of linear attenuation of radiation; γ , albedo of a single scattering; I, intensity of the radiation; $d\Omega$, element of the solid angle; σ , Stefan-Boltzmann constant; S, running coordinate on the beam; S_0 , length of the beam; ω , unit vector of the direction of the radiation; D, transmission function of the radiation; $\tau_0 = kH$, optical thickness; $\langle q \rangle$, density of the longitudinal radiative flow averaged over the channel section; $\bar{\theta}$, mean dimensionless mass temperature of the medium over the channel section.

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ANALOGY BETWEEN TURBULENT MOMENTUM AND HEAT TRANSFER UNDER COMPLEX CONDITIONS

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The analogy between turbulent momentum and heat transfer under complex conditions, i.e., under the action of several perturbing factors on the flow, is extended for a broad range of variation of the Prandtl number.

Because the systems of differential equations for the mean characteristics of the turbulent boundary layer are not closed, empirical relationships must be utilized. As the number of perturbing factors increases, the setting up of such relationships becomes more and more difficult; moreover, the formulas obtained can be applied only for the conditions of the experiment performed. The complexity of the computation is aggravated by the fact that the Reynolds analogy between the turbulent momentum and heat transfer is spoiled [1, 2].

The question of a more general analogy between the momentum and heat transfer is considered below in the turbulent core of a stationary near-wall flow under complex flow conditions that permit simplification of the heat-exchange parameter analysis.

The basic dimensionless variables used to analyze forced turbulent heat transfer are the Reynolds Re, Stanton St, or Nusselt Nu numbers, and the friction coefficient $c_f/2$, which are represented for convenience in the subsequent analysis as

$$\operatorname{Re} = u_{\delta}^{+} \delta^{+}, \tag{1}$$

$$St = \frac{1}{u_{\delta}^+ \, \vartheta_{\delta}^+} \,, \tag{2}$$

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